



c. If  $u = x^2 - y^2$ ,  $v = 2xy$  and  $x = r \cos\theta$ ,  $y = r \sin\theta$ , find  $\frac{\partial(u, v)}{\partial(r, \theta)}$ . (06 Marks)

d. In estimating the cost of a pile of bricks measured as  $2m \times 15m \times 1.2m$ , the tape is stretched 1% beyond the standard length. If the count is 450 bricks to 1 cu.m and bricks cost Rs.530 per 1000, find the approximate error in the cost. (06 Marks)

3 a. Select the correct answer :

i)  $\int_0^{\pi/2} \sin^n x dx$  is equal to

A)  $\frac{n+1}{n} I_{n-2}$       B)  $\frac{n+1}{n} I_{n+2}$       C)  $\frac{n-1}{n} I_{n-1}$       D)  $\frac{n-1}{n} I_{n-2}$

ii)  $\int_0^{\pi/2} \sin^4 x \cos^2 x dx$  is equal to

A)  $\frac{1}{16}$       B)  $\frac{1}{32}$       C)  $\frac{\pi}{32}$       D)  $\frac{\pi}{4}$

iii) The curve  $y^2(2a - x) = x^3$  is symmetrical about the

A) y - axis      B) x - axis      C) x and y axis      D) None of these.

iv) The asymptote for the curve  $r = a \sin 3\theta$  is equal to

A)  $\theta = a$       B)  $\theta = 3\theta$       C)  $\theta = 0$       D) No asymptotes.

(04 Marks)

b. Using the reduction formula, evaluate  $\int \tan^6 x dx$ .

(04 Marks)

c. If  $n$  is a positive integer, show that  $\int_0^{2\pi} x^n \sqrt{2ax - x^2} dx = \frac{(2n+1)!}{(n+2)!n!} \frac{a^{n+2}}{2^n} \pi$

(06 Marks)

d. Trace the Lemniscate  $a^2 y^2 = x^2 (a^2 - x^2)$

(06 Marks)

4 a. Select the correct answer :

i) Area bounded by the curve  $r = f(\theta)$  and the radii vectors  $\theta = \alpha$ ,  $\theta = \beta$  is

A)  $\frac{1}{2} \int r^2 d\theta$       B)  $\frac{1}{2} \int r^3 d\theta$       C)  $\frac{1}{2} \int_{\alpha}^{\beta} r^2 d\theta$       D)  $\frac{1}{2} \int_{\alpha}^{\beta} r^3 d\theta$

ii) The length of the arc of the curve  $y = f(x)$  between the points where  $x = a$  and  $x = b$  is

A)  $\int_a^b \left[ 1 + \left( \frac{dy}{dx} \right)^2 \right] dx$       B)  $\int_a^b \sqrt{1 - \left( \frac{dy}{dx} \right)^2} dx$       C)  $\int_a^b \sqrt{1 + \left( \frac{dy}{dx} \right)^2} dx$       D)  $\int_a^b \sqrt{1 + \left( \frac{dy}{dx} \right)^2} dx$

iii) The surface area of the solid generated by the revolution about x-axis of the arc of the curve  $y = f(x)$  from  $x = a$  to  $x = b$  is

A)  $\int_{x=a}^{x=b} 2\pi y ds$       B)  $\int_{x=a}^{x=b} 2\pi y dx$       C)  $\int_{x=a}^{x=b} 2\pi x ds$       D)  $\int_{x=a}^{x=b} 2\pi ds$

iv)  $\frac{d}{d\alpha} \left[ \int_a^b f(x, \alpha) dx \right]$  is equal to

A)  $\int_a^b \frac{\partial}{\partial \alpha} f(x, \alpha) dx$       B)  $\int_b^a \frac{d}{dx} f(x, \alpha) dx$       C)  $\int_a^b \frac{d}{d\alpha} f(x, \alpha) dx$       D)  $\int_a^b \frac{\partial}{\partial \alpha} f(x, \alpha) dx$  (04 Marks)

b. Find the entire length of the asteroid  $x^{2/3} + y^{2/3} = a^{2/3}$ , using the value of  $\frac{dy}{dx} = \frac{y^{1/3}}{x^{1/3}}$ .

(04 Marks)

- c. Find the volume of the solid generated by the revolution of the cardioid  $r = a(1 + \cos\theta)$  about the initial line. (06 Marks)
- d. Evaluate  $\int_0^1 \frac{x^\alpha - 1}{\log x} dx$ ,  $\alpha \geq 0$ . (06 Marks)

**PART - B**

5 a. Select the correct answer :

i) The order of the equation  $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = c^2 \left(\frac{d^2y}{dx^2}\right)^2$  is

- A) 1                      B) 2                      C) 3                      D) None of these.

ii) The standard form of a linear differential equation of the first order is

- A)  $\frac{dy}{dx} + y = P$       B)  $\frac{dy}{dx} + Py = Q$       C)  $\frac{dy}{dx} - Py = P$       D)  $\frac{dy}{dx} + Qy = Q$

iii) What is the value of  $\frac{\partial M}{\partial y}$ , for the differential equation

$$(1 + 2xy \cos x^2 - 2xy)dx + (\sin x^2 - x^2)dy = 0$$

- A)  $2x \cos x^2 - 2x$       B)  $2y \cos x^2 - 2x$       C)  $2x \cos x^2 - 2y$       D)  $-2x \cos x^2 - 2x$

iv) The differential equation of the family  $y^2 = 4a(x + a)$  is

- A)  $y^2 = \frac{dy}{dx} \left(x + \frac{1}{2}y \frac{dy}{dx}\right)$       B)  $y^2 = y \frac{dy}{dx} \left(x + \frac{1}{2}y \frac{dy}{dx}\right)$   
 C)  $y^2 = 2y \frac{dy}{dx} \left(x + \frac{1}{2}y \frac{dy}{dx}\right)$       D)  $y^2 = 2y \frac{dy}{dx} \left(x + y \frac{dy}{dx}\right)$  (04 Marks)

b. Solve  $dy/dx = e^{3x-2y} + x^2e^{-2y}$  (04 Marks)

c. Solve  $\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$  (06 Marks)

d. Find the orthogonal trajectories of the family of confocal conics  $\frac{x^2}{a^2} + \frac{y^2}{b^2 + \lambda} = 1$ , where  $\lambda$  is the parameter. (06 Marks)

6 a. Select the correct answer :

i) The series  $\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \dots$  converges if

- A)  $P > 0$                       B)  $P < 1$                       C)  $P > 1$                       D)  $P \leq 1$ .

ii) In a positive term series  $\sum u_n$ , if  $\lim_{n \rightarrow \infty} \frac{u_{n+1}}{u_n} = \lambda$ , then the series diverges for

- A)  $\lambda > 1$                       B)  $\lambda < 1$                       C)  $\lambda = 1$                       D)  $\lambda \leq 1$ .

iii) The  $n^{\text{th}}$  term of the series  $\left(\frac{2^2}{1^2} - \frac{2}{1}\right)^{-1} + \left(\frac{3^3}{2^3} - \frac{3}{2}\right)^{-2} + \left(\frac{4^4}{3^4} + \frac{4}{3}\right)^{-3} + \dots \infty$  is

- A)  $\left[\frac{(n+1)^n}{n^{n+1}} - \frac{n+1}{n}\right]^{-n}$       B)  $\left[\frac{(n+1)^{n+1}}{n^{n+1}} + \frac{n+1}{n}\right]^{-n}$       C)  $\left[\frac{(n+1)^{n+1}}{n^{n+1}} - \frac{n+1}{n}\right]^n$       D)  $\left[\frac{(n+1)^{n+1}}{n^{n+1}} - \frac{n+1}{n}\right]^{-n}$

iv) The series  $\frac{2}{1^2} - \frac{3}{2^2} + \frac{4}{3^2} - \frac{5}{4^2} + \dots$  is

- A) Conditionally convergent                      B) Absolutely convergent  
 C) Divergent                      D) None of the above. (04 Marks)



USN

--	--	--	--	--	--	--	--	--	--

10MAT11

**First Semester B.E. Degree Examination, June/July 2011**  
**Engineering Mathematics - I**

Time: 3 hrs.

Max. Marks:100

- Note:** 1. Answer any FIVE full questions, choosing at least two from each part.  
2. Answer all objective type questions only on OMR sheet page 5 of the answer booklet.  
3. Answer to objective type questions on sheets other than OMR will not be valued.

**PART - A**

- 1 a. i) If  $y = (ax + b)^{-1}$ , then  $y^n$  is (04 Marks)

A)  $\frac{(-1)^{n-1}(n-1)!a^n}{(ax + b)^n}$     B)  $\frac{(-1)^n n! a^n}{(ax + b)^{n+1}}$     C)  $\frac{n! a^n}{(ax + b)^{n+1}}$     D) Zero

- ii) The Taylor's theorem relates the value of the function and its  
A) I<sup>st</sup> order derivative    B) II<sup>nd</sup> order derivatives  
C) Constant    D) Higher order derivatives

- iii) Cauchy's mean value theorem reduces to Lagrange's mean value theorem, if  
A)  $f(x) = g(x)$     B)  $f(c) = g'(c)$     C)  $g(x) = x$     D)  $f(x) = 0$

- iv) To find the  $n^{\text{th}}$  derivative of a function  $y = f(x)$ , its  $(n-1)$  derivatives must be a  
A) function of  $y$     B) function of  $x$   
C) constant    D) function of  $x$  &  $y$

b. If  $\cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{n}\right)^n$  Prove that  $x^2 y_{n+2} + (2n + 1) x y_{n+1} + 2n^2 y_n = 0$  (06 Marks)

- c. Verify Lagrange's mean value theorem for the function  $f(x) = \log x$  in the interval  $[1, 2]$  and find the value of 'C'. (04 Marks)

- d. Expand  $\tan x$  in powers of  $(x - \pi/4)$  upto third degree term. (06 Marks)

- 2 a. i) L Hospital's rule implies that each differentiation reduces the order of the infinitesimals by  
A) unity    B) two    C) zero    D) four

- ii) If two curve cuts orthogonally, then angle between their tangents is equal to  
A) zero    B)  $\pi/4$     C)  $3\pi/4$     D)  $\pi/2$

- iii) Perpendicular distance from the pole on the tangent is equal to  
A)  $\sin \phi$     B)  $\cos \phi$     C)  $r \sin \phi$     D)  $r \cos \phi$

- iv) The value of radius of curve remains unchanged under the change of  
A) ordinates    B) signs    C) derivatives    D) none of these (04 Marks)

b. Evaluate  $\lim_{x \rightarrow 0} \left( \frac{1}{x} - \cot x \right)$ . (04 Marks)

- c. Prove that the radius of curvature  $\rho$  at any point  $(x, y)$  on the curve  $\sqrt{\frac{x}{a}} + \sqrt{\frac{y}{b}} = 1$  is given by

$$\rho = \frac{2(ax + by)^{3/2}}{ab}$$
 (06 Marks)

- d. Find the pedal equation of the curve  $\frac{2a}{r} = (1 + \cos \theta)$ . (06 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.  
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

- 3 a. i) A partial increment corresponds to a change of one of the variables and all other variables are \_\_\_\_\_ (04 Marks)  
 A) constant                      B) varying                      C) incremented                      D) decremented
- ii) If  $x = r \cos \theta$ ,  $y = r \sin \theta$ , the Jacobian of  $(x, y)$  with respect to  $(r, \theta)$  is equal to  
 A)  $\frac{1}{r}$                       B)  $\theta$                       C)  $r$                       D) zero
- iii) The necessary conditions for  $f(x, y)$  to have a maximum or minimum at  $(a, b)$ .  
 A)  $f_x(a, b) = 0$                       B)  $f_y(a, b) = 0$   
 C)  $f_{xy}(a, b) = 0$                       D)  $f_x(a, b) = f_y(a, b) = 0$
- iv) In Lagrange's method of undetermined multipliers are cannot determine the nature of the  
 A) Function                      B) Stationary point                      C) Multipliers                      D) None of these
- b. Find the extreme value of the function  $f(x) = x^3 + y^3 - 3axy$ ,  $a > 0$ . (06 Marks)
- c. If  $u = \log(x^3 + y^3 + z^3 - 3xy)$  show that  $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}$  (06 Marks)
- d. Find the percentage error in the area of an ellipse when an error of 1% is made in measuring the major and minor axis. (04 Marks)
- 4 a. i) Any motion in which the curl of the velocity vector is zero is said to be \_\_\_\_\_ (04 Marks)  
 A) rotational                      B) solenoidal                      C) irrotational                      D) conservative
- ii) The directional derivative of a scalar function  $\phi$  at any point is \_\_\_\_\_ along  $\nabla \phi$ .  
 A) minimum                      B) maximum                      C) zero                      D)  $\infty$
- iii) Gradient of a scalar field is a  
 A) constant                      B) scalar                      C) vector                      D) None of these
- iv) If  $\phi(x, y, z) = c$  is the equation of surface, then  $\nabla \phi$  is \_\_\_\_\_ to the surface.  
 A) parallel                      B) normal                      C) inclined                      D) not parallel
- b. Find the constants  $a, b, c$  so that the vector function  
 $\hat{F} = (x + 2y + az) \hat{i} + (bx - 3y - z) \hat{j} + (4x + (y + 2z) \hat{k}$  is irrotational (04 Marks)
- c. Prove that  $\text{grad div } F = \text{curl curl } F + \nabla^2 F$ . (06 Marks)
- d. Show that the spherical co-ordinate system is orthogonal. (06 Marks)

**PART - B**

- 5 a. i) Any integral formula which express in terms of another similar integral in lower powers is called \_\_\_\_\_ formula (04 Marks)  
 A) integral                      B) differential                      C) reduction                      D) trigonometric
- ii) If given equation contains only even powers of  $x$ , then the curve is symmetrical about  
 A)  $y$ -axis                      B)  $x$ -axis                      C) both axis                      D) None of these
- iii) Surface of solid generated by revolution about  $x$ -axis of the curve  $y = f(x)$  between  $x = a, x = b$ .  
 A)  $\int \pi y^2 dx$                       B)  $\int \pi x dy$                       C)  $\int \pi r^2 d\theta$                       D)  $\int 2\pi y ds$
- iv) Leibniz's rule for differentiation under integral sign is  
 A)  $\phi'(y) = \int \frac{\partial}{\partial y} f(x, y) dx$                       B)  $\phi'(y) = \int \frac{\partial}{\partial x \partial y} f(x, y) dx$   
 C)  $\phi(y) = \int \frac{\partial}{\partial x} f(x, y) dx$                       D) None of these
- b. Obtain the reduction formula for  $\int \cos^n x dx$ . (06 Marks)

c. Evaluate  $\int_0^{\pi/2} \sin^7 \theta \cos^6 \theta \, d\theta$ . (04 Marks)

d. Find the volume of the solid obtained by revolving the Astraid  $x^{2/3} + y^{2/3} = a^{2/3}$  about x - axis. (06 Marks)

6 a. i) The degree of the differential equation  $\frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{dx^2} = C$  is (04 Marks)

- A) Two                                      B) Three                                      C) One                                      D) Zero

ii) Variable separable form of the equation  $\frac{y \, dy}{x \, dx} = \sqrt{1+x^2+y^2+x^2y^2}$  is

A)  $\frac{\sqrt{1+y^2}}{y} \, dy = \frac{\sqrt{1+x^2}}{x} \, dx$                                       B)  $\frac{y}{\sqrt{1+y^2}} \, dy = x\sqrt{1+x^2} \, dx$

C)  $\sqrt{1+x^2} \, dx + \sqrt{1+y^2} \, dy$                                       D)  $\frac{y}{\sqrt{1+y^2}} \, dy = \frac{x}{\sqrt{1+x^2}} \, dx$

iii) The integrating factor of the differential equation  $x \log x \frac{dy}{dx} + y = \log x^2$  is

- A)  $\log x^2$                                       B)  $\log x$                                       C)  $x \log x$                                       D)  $x \log x^2$ .

iv) The differential equation  $(x + x^8 + ay^2) \, dx + (y^8 - y + bxy) \, dy = 0$  is exact if  $b =$  \_\_\_\_\_

- A) 4                                      B) 3x                                      C) 2a                                      D) 4a

b. Solve  $\frac{dy}{dx} = \frac{y}{x - \sqrt{xy}}$ . (04 Marks)

c. Solve  $\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$ . (06 Marks)

d. Find the orthogonal trajectories of the family of curve  $r^n \cos n\theta = a^n$ . (06 Marks)

7 a. i) The normal form of the matrix of rank r is (04 Marks)

A)  $\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$                                       B)  $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$                                       C)  $\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$                                       D)  $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

ii) If rank of the coefficient matrix is equal to rank of the Augmented matrix then equations are

- A) consistence                                      B) inconsistency  
C) have no solution                                      D) have infinite number of solutions.

iii) In Gauss - elimination method coefficient matrix reduces to \_\_\_\_\_ matrix.

- A) diagonal                                      B) unit matrix                                      C) triangular                                      D) None of these

iv) The system of linear homogeneous equations have trivial solution if all variable are (i = 1 ... n)

- A)  $x_i > 0$                                       B)  $x_i < 0$                                       C)  $x_i = 0$                                       D)  $x_i = \infty$

b. Investigate the value of  $\lambda$  and  $\mu$  so that the equations  $2x + 3y + 5z = 9$ ,  $7x + 3y - 2z = 8$ ,  $2x + 3y + \lambda z = \mu$  have i) unique solution ii) no solution iii) infinite number of solutions. (06 Marks)

c. Solve using the Gauss – Jordan method,

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 2 & -3 \\ -2 & -4 & -4 \end{bmatrix}$$

(06 Marks)

d. Find the rank of the Matrix of  $A = \begin{bmatrix} 1 & 2 & 4 & 3 \\ 2 & 4 & 6 & 8 \\ 4 & 8 & 12 & 16 \\ 1 & 2 & 3 & 4 \end{bmatrix}$

(04 Marks)

- 8 a. i) Each eigen vector corresponding to a eigen value is (04 Marks)  
 A) unique B) no unique C) infinite D) None of these  
 ii) The sum of the eigen values of the matrix is the sum of the elements of  
 A) Any row B) Any column  
 C) diagonal D) Any row and column.  
 iii) A homogeneous expression of the second degree in any number of variables is called  
 A) linear form B) cubic form C) quadratic form D) None of these  
 iv) Every square matrix satisfies its own \_\_\_\_\_ equation.  
 A) quadratic B) cubic C) algebraic D) characteristic
- b. Find the characteristics equation and eigen vector of the matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$

(06 Marks)

c. Reduce the matrix  $A = \begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$  to the diagonal form using characteristic equation method.

(06 Marks)

d. Reduce the quadratic form  $3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2zy$  to the canonical form.

(04 Marks)

\*\*\*\*\*